Code No: 111AL **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year Examinations, September - 2023
MATHEMATICAL METHODS
(Common to EEE, ECE, CSE, IT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Show that $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$. [2]

b) Show that $\delta^2 y_5 = y_1 - 2y_5 + y_4$. [3]

c) Find a root of the equation $x\, e^x = 1$, lying between 0 and 1 using the bisection method. [2]

d) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule taking $h = \frac{1}{4}$. [3]

e) Find the value of the constant term in the fourier series expansion of the function $f(x) = x$ in $(0,2\pi)$, $f(x+2\pi) = f(x)\ \forall x \in \mathbb{R}$. [2]

f) State the change of scale property for the Fourier Transform. [3]

g) Form the partial diferential equation by eliminating the arbitrary constants $z = a\,e^{-b^2 x}\cos by$; a, b are the arbitrary constants. [2]

h) Form the partial diferential equation by eliminating the arbitrary function [3]

$$z = x^2 f(x-y)$$

i) If $r^2 = x^2 + y^2 + z^2$ then find $\nabla.\frac{\vec{r}}{r}$. [2]

j) Find the curl(grad f), where f is a scalar function $2x^2 - 3y^2 + 4z^2$. [3]

PART - B

(50 Marks)

2.a) The points $(2,2), 5,4), (6,6)\ , (9,9)\ and\ (11,10)$ should be approximated by a straight line. Find that line.

b) Using Newton's forward difference formula, find a cubic polynomial for the following data. [5+5]

x	0	1	2	3	4	5
y	-3	3	11	27	57	107

OR

3.a) Fit a polynomial to the data given below and predict $f(1.5)$.

x	0	1	2	3
y	1	-1	-1	0

b) Fit a parabola to the following data. [5+5]

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

4.a) Solve $x\log_{10} x = 1.2$, using Regular falsi method.

b) Solve by the Euler's method the Initial value problem $\frac{dy}{dx} = \frac{x-y}{2}$, $y(0) = 1$ over [0, 3] step size =1/2. [5+5]

OR

5.a) Solve the system of equations

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Using the Gauss seidel iteration method. Use the initial approximations as $x_i = 0,\ i = 1, 2, 3$. Perform five iterations.

b) Given $y' = x^3 + y,\ y(0) = 2$ compute $y(0.2), y(0.4)\ and\ y(0.6)$ using the Runge-Kutta method of fourth order. [5+5]

6.a) Is $f(x) = \begin{cases} -\frac{1}{2}(\pi + x) & for\ \ -\pi \le x < 0 \\ \frac{1}{2}(\pi - x) & for\ \ 0 < x \le \pi \end{cases}$ even? If so, find the Fourier series for the function.

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

b) Find the Fourier series for $f(x) = 2lx - x^2$ in $0 < x < 2l$, $f(x + 2l) = f(x) \forall x \in \mathbb{R}$. [5+5]

OR

7.a) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & if\ \ |x| < a \\ 0\ , if & |x| > a \end{cases}$. Hence show that

$$\int_0^\infty \frac{\sin x - \cos x}{x^3} dx = \frac{\pi}{4}.$$

b) Find the inverse Fourier Sine Transform $f(x)$ of $F_s\{p\} = \frac{p}{1+p^2}$. [5+5]

8. If $p = \frac{\partial z}{\partial x}\ and\ q = \frac{\partial z}{\partial y}$, Solve the following partial differential equations

a) $p - q = \log(x + y)$

b) $y^2 p - xyq = x(z - 2y)$. [5+5]

OR

9. If $p = \frac{\partial z}{\partial x} and\ q = \frac{\partial z}{\partial y}$, solve the following partial differential equations

a) $\sin px \cos y = \cos px \sin y + p$

b) $p = \log(px - y)$. [5+5]

10.a) Prove that $\nabla(f^n) = nf^{n-1}\nabla f$.

b) For what points $P(x, y, z)$ does ∇f with $f = 25x^2 + 9y^2 + 16\,z^2$ have the direction from P to the origin? [5+5]

OR

11.a) Evaluate $\int_C \mathbf{F}(\mathbf{r}) \cdot \mathbf{dr}$ around the boundary C of the region R by Green's Theorem, where, $\mathbf{F} = grad(x^3 \cos^2(xy))$ and $R: 1 \le y \le 2 - x^2$.

b) Evaluate $\oint_C\ e^2 dx + 2y\,dy - dz$ by Stoke's theorem where 'c' is the curve $x^2 + y^2 = 4,\ z = 2$. [5+5]

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